

MATH 521A: Abstract Algebra Exam 2

Please read the following instructions. For the following exam you are free to use a calculator and any papers you like, but no books or computers. Please turn in **exactly six** problems. You must do problems 1-4, and two more chosen from 5-8. Please write your answers on separate paper, make clear what work goes with which problem, and put your name or initials on every page. You have 75 minutes. Each problem will be graded on a 5-10 scale (as your quizzes), for a total score between 30 and 60. This will then be multiplied by $\frac{5}{3}$ for your exam score.

Turn in problems 1,2,3,4:

1. Determine, with proof, all zero divisors in \mathbb{Z}_{34} . How many are there?
2. Find all solutions to the modular equation $50x \equiv 20 \pmod{630}$.
3. For ring R and element $x \in R$, we say that x is *silver* if $x + x + x = 0_R$. Define $T \subseteq R$ to be the set of silver elements of R . Prove that T is a subring of R .
4. Consider the function $f : \mathbb{Z}_{34} \rightarrow \mathbb{Z}_2 \times \mathbb{Z}_{17}$ given by $f : [x]_{34} \mapsto ([x]_2, [x]_{17})$. Prove that f is well-defined.

Turn in exactly two more problems of your choice:

5. Let R have ground set \mathbb{Z} and operations given by:

$$\forall x, y \in \mathbb{Z}, \quad x \oplus y = x + y - 2, \quad x \odot y = 2x + 2y - xy - 2.$$

Prove that R , with operations \oplus, \odot , is a commutative ring.

6. Let R be a (not necessarily commutative) ring with identity and $x, y \in R$. Suppose that neither x nor y is a zero divisor, and that xy is a unit. Prove that x is a unit.
7. Let R be the ring of 2×2 upper triangular matrices with entries from \mathbb{Q} , i.e. $R = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : a, b, c \in \mathbb{Q} \right\}$. Determine, with proof, all units and zero divisors of R .
8. Let R be the ring of 2×2 matrices with entries from \mathbb{Q} . Define $f : R \rightarrow R$ via $f : \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mapsto \begin{pmatrix} a & c \\ b & d \end{pmatrix}$, a.k.a. the matrix transpose. Prove or disprove that f is a ring isomorphism.

You may also turn in the following (optional):

9. Describe your preferences for your next group assignment. (will be kept confidential)